

Numerical coefficients of 0.91 and 0.95 were obtained in analogous expressions in [1, 2] respectively, with allowance for the pressure actually acting on the diaphragm. The model of a freely linked chain gives a value  $\sim 1.5$  times lower for the time of complete opening than the "hinge" model.

The relative value of the through cross section at different stages of the process of opening of the diaphragm calculated for the "hinge" model and the freely linked chain is shown in Fig. 3 {1) model of "hinge" opening; 2) inelastic opening; 3) experimental dependence for a thick copper diaphragm; 4) experiment of [3]}.

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#### TRANSIENT HEAT-MASS EXCHANGE NEAR A SPHERICAL PARTICLE

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##### 1. Statement of the Problem and the Basic Equations

The spherically symmetric problem is discussed in which phase transitions occur only on the surface of a particle, and the mass velocities which arise in the gas are many times smaller than the speed of sound. In this case it makes sense to use the condition of pressure uniformity over space (appropriate justification occurs in [1]). The gas which surrounds the drop or particle is a single-component gas and is the vapor of the material of the drop or particle (there is no diffusion in the system). Let the particle be incompressible, motion be absent in it, but thermal conductivity occur. We will assume the gas or vapor to be a perfect gas. The system of equations which describes this process has the form

$$r > r_0, p = p(t), p = \rho RT; \quad (1.1)$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v) = 0; \quad (1.2)$$

$$\rho \frac{\partial c_v T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \lambda_1 \frac{\partial T}{\partial r} \right) - \rho v \frac{\partial c_v T}{\partial r} - \frac{p}{r^2} \frac{\partial}{\partial r} (r^2 v); \quad (1.3)$$

$$r < r_0, \rho_2 = \frac{\partial c_2 T}{\partial t} = \frac{1}{r^2} \left( r^2 \lambda_2 \frac{\partial T}{\partial r} \right), \rho_2 = \text{const}, \quad (1.4)$$

where  $\rho$  is the density,  $T$  is the temperature,  $p$  is the pressure,  $v$  is the velocity,  $R$  is the gas constant,  $\lambda$  is the thermal conductivity coefficient,  $r$  is the radius, and  $t$  is the time. The subscripts 1 and 2 correspond to the values of the parameters in the vapor and in the particle, and the subscript  $\sigma$  corresponds to values on the surface of the particle.

The first of Eqs. (1.1) is the pressure uniformity condition over space, which is a corollary of the momentum equation upon neglect of inertial forces, (1.2) is the continuity equation in the gaseous phase, and (1.3) and (1.4) are the heat flux equations in the vapor and the particle, respectively.

It is possible to derive the relation

$$\frac{\gamma p}{r^2} \frac{\partial}{\partial r} (r^2 v) = \frac{\gamma - 1}{r^2} \frac{\partial}{\partial r} \left( r^2 \lambda_1 \frac{\partial T}{\partial r} \right) - \frac{dp}{dt} \quad (1.5)$$

from (1.1)-(1.3). Having integrated (1.5) along the coordinate with account taken of the fact that  $p$  and  $dp/dt$  are functions only of the time, we obtain

$$v = \frac{r_\sigma^2}{r^2} v_{1\sigma} + \frac{\gamma - 1}{\gamma p} \left[ \lambda_1 \frac{\partial T}{\partial r} - \frac{r_\sigma^2}{r^2} \lambda_{1\sigma} \left( \frac{\partial T}{\partial r} \right)_{1\sigma} \right] - \frac{r^3 - r_\sigma^3}{3r^2 \gamma p} \frac{dp}{dt}. \quad (1.6)$$

Thus within the framework of a model uniform with respect to the pressure the velocity distribution in the vapor is uniquely specified at each instant of time by the temperature distribution in it and by the values of the velocities at the two radii. One of the velocity values can be taken at the surface of the particle, and the other value determines  $dp/dt$ .

Let us select

$$t = 0, r > r_{\sigma 0}, T = T_{10}, p = p_0, r < r_{\sigma 0}, T = T_{20} \quad (1.7)$$

as the initial conditions, where the subscript 0 corresponds to the initial state of the system.

The boundary conditions on the surface of a particle or drop are of the form

$$\begin{aligned} r = r_\sigma, \rho_{1\sigma}(v_{1\sigma} - dr_\sigma/dt) = j, j = -\rho_2(dr_\sigma/dt), \\ \lambda_{2\sigma}(\partial T/\partial r)_{2\sigma} = \lambda_{1\sigma}(\partial T/\partial r)_{1\sigma} - lj, T_{1\sigma} = T_{2\sigma} = T_S(p), \end{aligned} \quad (1.8)$$

where  $j$  is the intensity of the phase transitions referred to unit surface and time and  $l$  is the heat of vaporization; the last of the conditions (1.8) is the phase equilibrium condition on the surface of the particle. This equilibrium may be absent in the entire remaining volume [ $T \neq T_S(p)$ ]. In the case of very rapid explosive processes phase equilibrium may not be fulfilled on the surface of the particle when the phase transitions on it do not have time to follow the variation of the pressure. In this case it is necessary to include the kinetic relation

$$j \sim F|T_{1\sigma} - T_S(p)|. \quad (1.9)$$

When  $F \rightarrow \infty$  the finiteness of  $j$  is ensured in the case of the equilibrium conditions  $T_{1\sigma} \rightarrow T_S(p)$  adopted in (1.8).

If there are no phase transitions on the interphase surface, the boundary conditions on it are simplified to

$$r = r_\sigma, v_{1\sigma} = 0, \lambda_{1\sigma} \left( \frac{\partial T}{\partial r} \right)_{1\sigma} = \lambda_{2\sigma} \left( \frac{\partial T}{\partial r} \right)_{2\sigma}, T_{1\sigma} = T_{2\sigma}.$$

One should specify the other boundary condition on the outer boundary of the system (in the vapor). Let us consider the case in which the particle is placed at the center of a spherical volume of radius  $r_c$  filled with vapor with the boundary conditions cited below and the equation for the rate of pressure variation which follows from them:

$$\begin{aligned} r \neq r_c, \frac{\partial T}{\partial r} = 0, \rho v = 0, \\ \frac{dp}{dt} = \frac{3r_\sigma^2}{r_c^3 - r_\sigma^3} \left[ \gamma p v_{1\sigma} - (\gamma - 1) \lambda_{1\sigma} \left( \frac{\partial T}{\partial r} \right)_{1\sigma} \right]. \end{aligned} \quad (1.10)$$

These conditions correspond to a uniform "cell" model of a dispersed mixture, according to which a spherical volume of radius  $r_c$  concentric with the particle corresponds to each particle of radius  $r_{c0}$ . The cell radius  $r_c$  is determined either by the number of particles per unit volume of the mixture  $n$ , or by the volume content of particles in the system  $\alpha_2$ , or the mass vapor content  $x_1$ :

$$n = \frac{3}{4\pi r_c^3}, \quad \alpha_2 = \frac{r_{c0}^3}{r_c^3}, \quad x_1 = \frac{(r_c^3 - r_{c0}^3)\rho_{i0}}{(r_c^3 - r_{c0}^3)\rho_{i0} + r_{c0}^3\rho_2}.$$

It is assumed that the disturbances introduced into the vapor by each particle are localized inside the cell corresponding to it. If one assumes the absence of a variation of the volume and the energy of the entire mixture, the identical nature of all the particles and the cells corresponding to them, as well as the absence of mass and energy exchange among them, then we obtain the conditions (1.10) on the boundaries of the cells. If the cell radius tends to infinity ( $r_c \rightarrow \infty$ ,  $d_2 \rightarrow 0$ ) and we maintain constant parameters on the external boundary, then the pressure in the vapor will be constant and

$$r_c = \infty, \quad T(t) = T_0, \quad p(t) = p_0. \quad (1.11)$$

The boundary conditions (1.11) correspond to the case in which a single particle is placed in an infinite volume and mass of vapor.

Thus, the problem reduces to the solution of the system of equations (1.1), (1.3), (1.4), and (1.6) with the boundary conditions (1.8), (1.10), or (1.11) and the initial conditions (1.7). The system is closed by the specification of the functions  $\lambda_1$ ,  $c_V$ ,  $\lambda_2$ , and  $c_2$ . Since these functions vary little in the temperature intervals under discussion, we will assume that  $\lambda_1$  is a linear function of the temperature and  $\lambda_2$ ,  $c_V$ , and  $c_2$  are constants.

Let us introduce the following dimensionless parameters:

$$\begin{aligned} \xi &= r/r_{c0}, \quad \xi_\sigma = r_\sigma/r_{c0}, \quad \tau = \kappa_{10}t/r_{c0}^2, \quad U = r_{c0}v/\kappa_{10}, \\ P &= p/p_0, \quad \Theta_{20} = T_{20}/T_{10}, \quad \Theta = T/T_{10}, \quad \Theta_S = T_S/T_{10}, \\ \delta &= \rho_2/\rho_{10}, \quad L = l/c_p T_{10}, \quad \delta_2 = \rho_2/\rho_{10}, \quad J = -\delta_2 d\xi_\sigma/d\tau, \\ \Lambda_1 &= \lambda_1/\lambda_{10}, \quad \Lambda_2 = \lambda_2/\lambda_{10} \quad (\kappa_{10} = \lambda_{10}/\rho_{10}c_p, \quad \kappa_2 = \lambda_2/\rho_2c_2), \end{aligned}$$

where  $\kappa_{10}$  and  $\kappa_2$  are the thermal-conductivity coefficients of the gas and the particle.

The system of equations, along with the initial and boundary conditions, will take the form

$$\begin{aligned} \xi > \xi_\sigma, \quad P(\tau) &= \delta\Theta, \\ \frac{\partial\Theta}{\partial\tau} &= \frac{1}{\xi^2\delta} \frac{\partial}{\partial\xi} \left( \xi^2 \Lambda_1 \frac{\partial\Theta}{\partial\xi} \right) - U \frac{\partial\Theta}{\partial\xi} + \frac{\gamma-1}{\gamma\delta} \frac{dP}{d\tau}, \end{aligned} \quad (1.12)$$

$$\begin{aligned} U &= \frac{\xi_\sigma^2}{\xi^2} U_{1\sigma} + \frac{1}{P} \left[ \Lambda_1 \frac{\partial\Theta}{\partial\xi} - \frac{\xi_\sigma^2}{\xi^2} \Lambda_{1\sigma} \left( \frac{\partial\Theta}{\partial\xi} \right)_{1\sigma} \right] - \frac{\xi^3 - \xi_\sigma^3}{3\xi^2\gamma P} \frac{dP}{d\tau}; \\ \xi < \xi_\sigma, \quad \frac{\partial\Theta}{\partial\tau} &= \frac{\kappa_2}{\kappa_{10}} \frac{1}{\xi^2} \frac{\partial}{\partial\xi} \left( \xi^2 \frac{\partial\Theta}{\partial\xi} \right); \end{aligned} \quad (1.13)$$

$$\tau = 0, \quad \Theta_1 = 1, \quad \Theta_2 = \Theta_{20}, \quad P = P_0 = 1; \quad (1.14)$$

$$\xi = 0, \quad \partial\Theta/\partial\xi = 0; \quad (1.15)$$

$$\delta_{1\sigma} \left( U_{1\sigma} - \frac{\partial\xi_\sigma}{\partial\tau} \right) = J, \quad \Theta_{1\sigma} = \Theta_{2\sigma} = \Theta_S(P), \quad (1.16)$$

$$\begin{aligned} \Lambda_2 \left( \frac{\partial\Theta}{\partial\xi} \right)_{2\sigma} &= \Lambda_{1\sigma} \left( \frac{\partial\Theta}{\partial\xi} \right)_{1\sigma} - LJ; \\ \xi &= \xi_\sigma, \quad \partial\Theta/\partial\xi = 0, \quad \delta U = 0, \end{aligned} \quad (1.17)$$

$$\frac{dP}{d\tau} = \frac{3\xi_\sigma^2\gamma}{\xi^3 - \xi_\sigma^3} \left[ PU_{1\sigma} - \Lambda_{1\sigma} \left( \frac{\partial\Theta}{\partial\xi} \right)_{1\sigma} \right].$$

in the new dimensionless variables.

The boundary conditions (1.16) are specified on the surface of the particle, whose radius  $r_G(t)$  or  $\xi_G(t)$  in the presence of phase transitions varies in time and should be determined in the course of the solution.

The condition (1.17) should replace the condition

$$\xi_c = \infty, \theta = 1, P = 1 \quad (1.17a)$$

for the case of an infinite cell.

In this connection a replacement of the variable  $\xi$  by  $\eta = 1/\xi$  was carried out in Eqs. (1.12) for the reduction of the infinite integration region  $\xi_G < \xi < \infty$  to a finite one.

As is evident from the system of equations (1.12) and (1.13) and the initial and boundary conditions (1.14)-(1.16) and (1.17a), the initial radius of the particle does not contribute to the number of controlling parameters for the case  $\xi_c = \infty$ . Due to this fact the dimensionless solution will be self-similar, i.e., identical for all particle sizes. This situation is associated, in particular, with the fact that the condition of phase equilibrium on the interphase surface  $T_G = T_S(p)$  was used in the statement of the problem. If one takes account of the nonequilibrium kinetics of phase transitions, assuming  $T_G \neq T_S(p)$ , then the initial radius of the particle and the characteristic time of the kinetics of nonequilibrium phase transitions (1.9) determined by the quantity  $F$  will enter into the number of controlling parameters.

The calculation was carried out on a computer according to an implicit scheme of first-order accuracy for the system of a drop of water in water vapor with the appropriate thermo-physical parameters [2].

## 2. Time-Independent Solution

The case in which a spherical particle is placed in an infinite volume of vapor ( $\xi_c = \infty$ ) is discussed. Its radius, regardless of phase transitions, does not vary. The temperature inside the particle is held uniform and constant, and the parameters at infinity are also kept constant. One can consider the time-independent solution of the formulated problem as the limit of the transient problem as  $\tau \rightarrow \infty$  and with the boundary conditions preserved. The processes taking place in the vapor are described by the system of equations (1.12), which can, setting all time derivatives equal to zero, be reduced to a single equation in  $\theta$ , namely,

$$\begin{aligned} \left( \frac{d\theta}{d\xi} \right)^2 - \frac{(1-\varepsilon)}{\xi^2} \left( \frac{d\theta}{d\xi} \right)_{1\sigma} \frac{d\theta}{d\xi} - \frac{\theta}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta}{d\xi} \right) &= 0, \\ U = \frac{d\theta}{d\xi} - \frac{(1-\varepsilon)}{\xi^2} \left( \frac{d\theta}{d\xi} \right)_{1\sigma}, \quad \left( \varepsilon = \frac{c_p T_S}{l} \left( 1 - \frac{\rho_{10}}{\rho_{20}} \right) \right), \\ \theta(1) = \theta_S, \theta(\infty) = 1. \end{aligned}$$

As a result of integration we obtain

$$\begin{aligned} \theta &= (d_2 e^{-d_1/\xi} - \beta)/d_1, \quad U = d_1 \theta / \xi^2, \\ d_1 &= \ln \left( 1 + \frac{1 - \theta_S}{\theta_S} \varepsilon \right), \quad d_2 = d_1 \left( 1 + \frac{1 - \varepsilon}{\varepsilon} \theta_S \right), \\ (d\theta/d\xi)_{1\sigma} &= d_1 \theta_S' \varepsilon, \quad \beta = (1 - \varepsilon)(d\theta/d\xi)_{1\sigma}. \end{aligned} \quad (2.1)$$

In the absence of phase transitions, when the solution is obtained for  $l = \infty$  and  $\varepsilon = 0$ , we have

$$\theta = 1 - (1 - \theta_S)/\xi, \quad U = 0.$$

In the time-independent problem under discussion the dimensionless heat flow at the surface of the particle (the Nusselt number) has the form

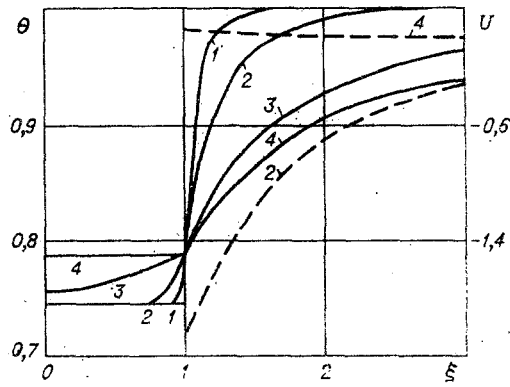


Fig. 1

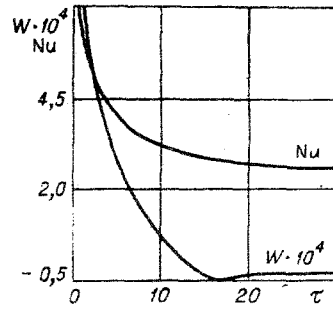


Fig. 2

$$Nu_{st} = 2 \ln [1 + \varepsilon(1 - \Theta_S)/\Theta_S] \varepsilon(1 - \Theta_S)/\Theta_S. \quad (2.2)$$

As a rule,  $\varepsilon = c_p T_S / l \ll 1$ ; therefore, one can simplify Eq. (2.2) as follows:

$$Nu_{st} = 2 - c_p(T_{10} - T_S)/l.$$

The limiting or time-independent Nusselt number is equal to 2 in the absence of phase transitions ( $l = \infty$ ) on the surface of the particle. In the opposite case, in which evaporation or blowing-in occurs ( $T_{10} > T_S$ ),  $Nu_{st} < 2$ , and when condensation or suction occurs ( $T_{10} < T_S$ ),  $Nu_{st} > 2$  on the surface of a sphere of constant radius.

The characteristic variation time of the particle size,

$$t_\sigma = 2\rho_{20}r_{\sigma 0}^2/3\lambda_{10}Nu_{st}\Delta T,$$

is much larger than the characteristic time of temperature equalization in the gaseous phase,  $t_1 = r_\sigma^2/\alpha_1$  ( $t_1/t_\sigma \approx 10^{-4}$ ). Therefore, one can consider the temperature distribution (2.1), which is obtained on the assumption that the particle radius does not vary, as the one corresponding to the instantaneous radius  $r_\sigma(t)$ .

Thus the solution (2.1) describes in practice the quasi-time-independent behavior of the system upon a variation of the particle radius occurring due to evaporation or condensation. The law of boundary movement is determined by the relation

$$dr_\sigma/dt = \lambda_{10}Nu_{st}(T_{10} - T_S)/2r_\sigma\rho_2l.$$

An analogous relation is cited in [3].

### 3. Results of the Solution

Alternatives were calculated with different initial temperature differentials in the phases and different ratios of the initial radius of the particle to the cell radius. Cases with zero intensity of the phase transitions were considered to illustrate the effect of phase transitions.

Three temperature conditions are discussed in the case of an initial pressure  $p_0 = 1$  bar  $T_{S0} = 373^\circ\text{K}$ , and two values of the mass vapor content  $x_1 = 1$  ( $\alpha_2 = 0$ ) and  $x_1 = 0.1$  ( $\alpha_2 = 0.8 \cdot 10^{-2}$ ):

I	$T_{10} = 473 \text{ K}, T_{20} = 353 \text{ K};$
II	$T_{10} = 373 \text{ K}, T_{20} = 293 \text{ K};$
III	$T_{10} = 473 \text{ K}, T_{20} = 378 \text{ K}.$

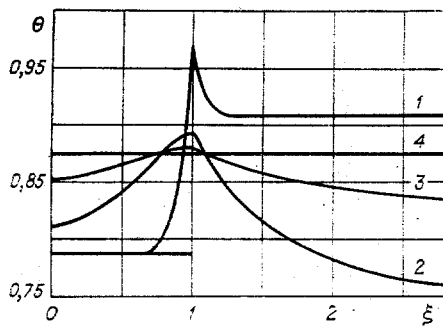


Fig. 3

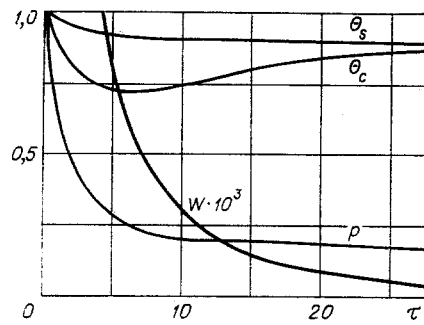


Fig. 4

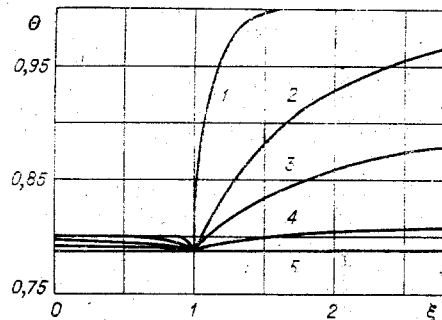


Fig. 5

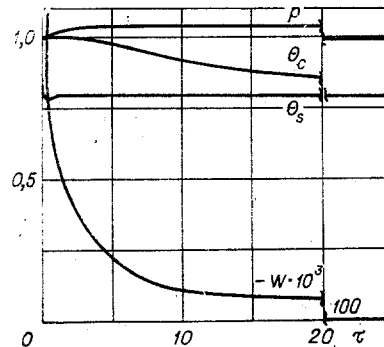


Fig. 6

The results of the solution for the conditions I in the case of  $x_1 = 1$  are presented in Figs. 1 and 2. Curves 1-4 (Fig. 1) represent the temperature distributions (solid lines) and velocity distributions (dashed lines), respectively, at the instants of time  $\tau = 0.01, 0.05, 10,$  and  $\infty$ . The pressure and temperature of the surface of the particle determined by the condition of phase equilibrium are kept constant. As  $\tau \rightarrow \infty$  the temperature and velocity curves tend to limiting or to quasi-time-independent configurations (2.1) in which the linear scale is determined by the instantaneous particle size  $r_0(t)$  and  $Nu \rightarrow Nu_{st}$ . The limiting temperature distribution in the gaseous phase is attained by an instant  $\tau \rightarrow 10$ . A uniform distribution is established later in the particle at  $\tau > 10\kappa_{10}/\kappa_2$  on account of the fact that  $\kappa_{10} > \kappa_2$ . The temperature and velocity distributions are monotonic at each instant of time. In the absence of phase transitions the velocity has an extremum, which is shifted away from the particle as the time increases. The variation of the Nusselt number (the dimensionless heat flow into the particle) and of the dimensionless phase transition rate  $W = J/\delta_2$  is shown in Fig. 2. Complete evaporation of the particle occurs in the case under discussion, but condensation of vapor precedes it at first. This circumstance is associated with the fact that at times  $t \ll r_{00}^2/\kappa_2$  only a narrow layer in the particle at its surface is heated up. This situation results in the appearance of large temperature gradients in the particle, so that  $LJ = \Lambda_{10}(\partial\theta/\partial\xi)_{10} - \Lambda_2(\partial\theta/\partial\xi)_{20} < 0$ . At  $\tau > 40$  the temperature distribution inside the particle becomes more mildly sloping, and condensation is replaced by evaporation.

In cases in which the particle is placed in a finite volume of vapor the solution is significantly altered. The main difference consists of the fact that the pressure in the vapor varies with time. In the presence of phase transitions the surface temperature also varies in agreement with the equilibrium condition  $T_0 = T_S(p)$ . The variations of the dimensionless temperature, pressure, and intensity of the phase transitions for conditions II in the case of  $x_1 = 0.1$  ( $\alpha_2 = 0.8 \cdot 10^{-2}$ ) are shown in Figs. 3 and 4. Condensation of the vapor results in an expansion of the remaining mass of vapor, due to which there occurs a significant cooling of it, which cannot at first be compensated by the heat liberated upon condensation (Fig. 3, in which curves 1-4 correspond to the instants of time  $\tau = 0.5, 10, 20,$  and  $\infty$ ). The temperature on the cell boundary drops down to  $269^\circ\text{K}$  (Fig. 4). Subsequently, the heat liberated upon condensation heats up the vapor. The temperatures of the particle and the vapor are equalized as  $\tau \rightarrow \infty$ , and the process is asymptotically discontinued. The temperature and velocity distributions at each instant of time are monotonic. In this case a significant pressure decrease (by approximately a factor of five) is obtained during a time of the order of  $r_{00}^2/\kappa_{10}$  (see Fig. 4), which indicates the effectiveness of even a small (with

respect to volume) injection of cold drops into the vapor in the case of an emergency pressure increase.

In the absence of phase transitions the vapor pressure also decreases when  $x_1 < 1$  and  $T_{10} > T_{20}$ , but just because of its cooling due to thermal conductivity.

The solution for the conditions III in the case of  $x_1 = 0.1$  ( $\alpha_2 = 0.8 \cdot 10^{-2}$ ) is presented in Figs. 5 and 6. Evaporation of the particle occurs in the system. The temperature curves 1-5 in Fig. 5 correspond to the instants of time  $\tau = 0.01, 2, 15, 50, \text{ and } \infty$ . The pressure in the system increases from  $p_0 = 1$  to  $p = 1.03$  bar and then decreases to  $p = 0.98$  bar. This decrease in the pressure is associated with the fact that phase transition is practically discontinued but heat exchange still occurs (see Fig. 6). We note that when  $x_1 < 1$  condensation is not always replaced by evaporation under conditions I.

The particle size varied little in all the alternatives discussed. In the case  $x_1 = 1$  (infinite volume of vapor) this circumstance is associated with the fact that the calculations were performed prior to the emergence into quasi-time-independent conditions. One can use the time-independent solution (2.1) to describe the subsequent behavior of the system, as has already been pointed out.

In the "cell" formulation the small variation of the radius prior to the instant of establishment of equilibrium is due to the fact that alternatives with a small vapor mass content in the cell,  $x_1 = 0.1$ , were discussed.

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#### DYNAMICS OF A CYLINDRICAL CAVITY IN A COMPRESSIBLE LIQUID

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The equation of one-dimensional pulsation of a cylindrical cavity in a compressible liquid was derived in [1, 2] within the framework of the approximate theory of Kirkwood-Bethe [3], which is based on the approximation by the function  $G = r^{1/2}\Omega$  of an invariant propagating along a characteristic at a velocity  $c + u$ , where  $\Omega = \omega + u^2/2$  is the kinetic enthalpy,

$\omega = \int dp/\rho$  is the enthalpy,  $u$  is the velocity of a fluid particle,  $r$  is the coordinate, and  $c$  is the local speed of sound.

In the derivation of this equation

$$\frac{\partial}{\partial t} [r^{1/2} (\omega + u^2/2)] = - (c + u) \frac{\partial}{\partial r} [r^{1/2} (\omega + u^2/2)] \quad (1)$$

the condition for  $G$  was used, as well as the continuity and momentum conservation equations, on the basis of which the replacement of partial derivatives by total ones was made in (1) [2]. The pulsation equation of the cavity is derived in the following form (we set  $r = R$ ,  $u = dR/dt$ ):

$$\begin{aligned} R[1 - (dR/dt)/c]d^2R/dt^2 + (3/4)(dR/dt)^2[1 - (dR/dt)/3c] = \\ = \omega[1 + (dR/dt)/c]/2 + R(d\omega/dt)[1 - (dR/dt)/c]/c, \end{aligned} \quad (2)$$